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Mass and Angular Momentum of Black Holes: An Overlooked Effect of General Relativity Applied to the Galactic Center Black Hole Sgr A*

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Abstract I report the discovery of a new effect of General Relativity which is important to understand very rapidly rotating (Kerr) black holes. The orbital velocity of a test particle is no longer a monotonic function of the orbit radius when the spin of the black hole is >0.9953, but displays a local minimum-maximum structure for radii smaller than 1.8 gravitational radii. There the rate of change of the orbital velocity per radius unit equals the radial epicyclic frequency and is exactly one third of the polar epicyclic frequency, suggesting a 3:1 resonant oscillatory motion of the particle. If associated with the most recently observed quasi-periods the mass of the supermassive black hole Sgr A* in the centre of the our Galaxy is determined to $3.3\times10^6 M_\odot$, and the spin is 0.99616.

Key words: Galaxy: center - X-rays: general - black hole physics - X-rays: individuals - Sgr A*

1 INTRODUCTION

The classical problem of a test particle orbiting a rotating black hole has long been solved (Bardeen et al. 1972). Stable circular orbits exist down to a minimal orbital radius r, the innermost marginally stable circular orbit.

As for Newtonian mechanics both the energy E and the angular momentum L of the particle are monotonic functions of r for the full range of the black hole spin parameter a, the normalized angular momentum for which -1$\leq a \leq$ 1. This means that there is no

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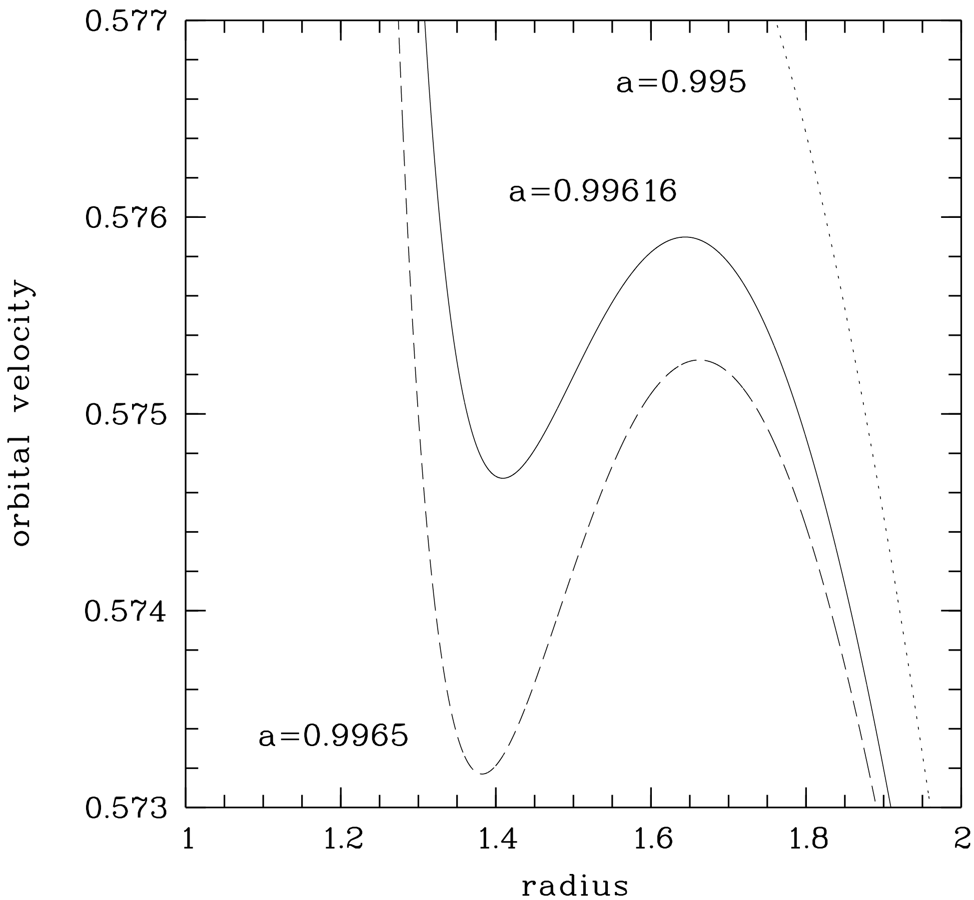

Fig. 1 Orbital velocity in units of c vs. orbital radius r in units of the gravitational radius for various black hole spins a.

obvious preference for any specific value of r and a for a particle to take. This strictly monotonic behaviour with r and a has generally been assumed to hold for the orbital velocity $v^{(\Phi)}$ as well. But this is not the case.

2 THE STRANGE BEHAVIOR OF THE ORBITAL VELOCITY

By a detailed numerical analysis of the Boyer-Lindquist functions (Boyer & Lindquist 1967), which describe the space-time of a Kerr black hole, I have shown that the monotonic behaviour of $v^{(\Phi)}$ breaks down for $a > 0.9953$, and $v^{(\Phi)}$ develops a minimum-maximum structure in r-space (c.f. Fig. 1) (Aschenbach 2004).

It can be shown that the radii with $r_{min} \leq r \leq r_{max}$ are larger than the corresponding radius of the innermost stable circular orbit (Aschenbach 2004). Therefore, according to the formal Bardeen stability criterion these orbits are stable (c.f. Fig. 2). The non-monotonic behaviour of $v^{(\Phi)}$ is a new effect of General Relativity which has been overlooked so far. Meanwhile the effect has been confirmed and Stuchlík et al. (2005) have shown that this effect occurs not only for geodesic, stable, circular orbits but also for

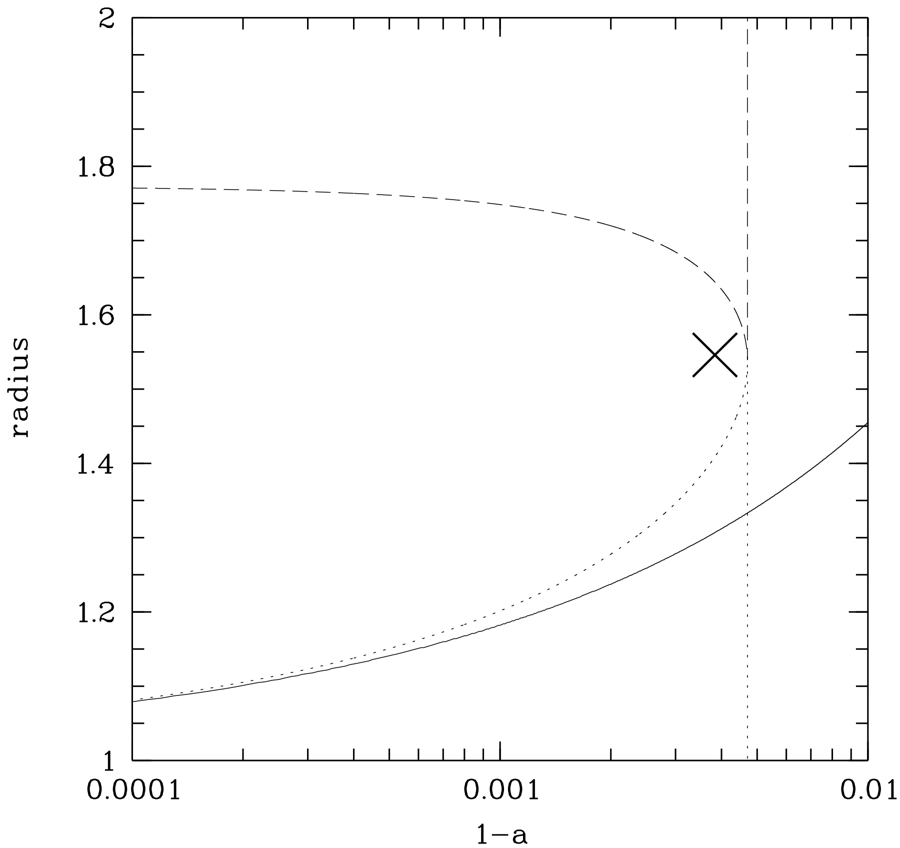

Fig. 2 Radii, in units of the gravitational radius, of the local maximum/minimum of the orbital velocity (curved dashed/dotted line) as function of $1 - a$. The solid line marks the radius of the innermost stable orbit. The vertical dashed line represents the critical spin $a = 0.9953$.

nongeodesic circular orbits with constant specific angular momentum (Stuchlík et al. 2005). The physical relevance and impact of this new effect are currently investigated. Variation of the topology for particles and fluids orbiting rapidly rotating black holes is a possible consequence (Stuchlík et al. 2005).

According to standard definition the orbits between r_{min} and r_{max} are stable but this region is suspicious because it is the only region with $\partial v^{(\Phi)}/\partial r \geq 0$, which represents a positive rate of change of the orbital velocity per unit length of r or some sort of time scale that can be compared with other typical time scales. These are given by the orbital (Kepler) frequency, the radial epicyclic frequency Ω_R and the vertical (polar) epicyclic frequency Ω_V. These three frequencies take identical values in Newtonian physics but all three differ from each other only in Kerr space-time for $a \neq 0$. I therefore define a critical angular 'frequency' $\Omega_c = 2\pi \frac{\partial v^{(\Phi)}}{\partial r} \mid_{max}$ at that radial position where $\partial v^{(\Phi)}/\partial r$ has the maximum value for a given a. Fig. 3 shows Ω_c in comparison with Ω_R and illustrates that Ω_c is very close to Ω_R for $1-a = 0.004$, i.e. the rate of change of the orbital velocity in radial direction equals the epicyclic frequency again in radial direction.

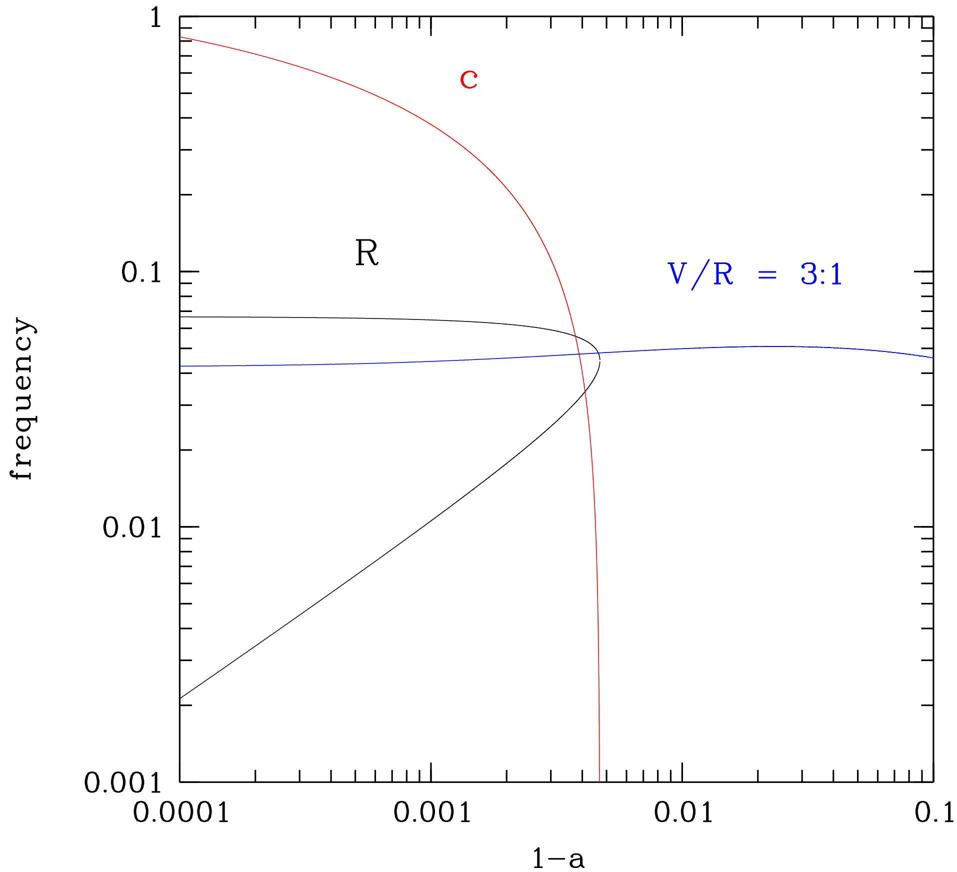

Fig. 3 Critical frequency (c), radial epicyclic frequency (R) at r_{min} and r_{max} (upper and lower branch of the asymmetric parabolic curve) and the vertical epicyclic frequency divided by a factor of three (V/R = 3:1) are shown; a common intersection occurs around $1 - a = 0.004$.

Any pertubation at this point in space-time may trigger and maintain radial quasi-periodic motions with Ω_R. Fig. 3 also shows that a common intersection shows up for $\Omega_V = 3 \times \Omega_R$. A 3:1 resonance exists between the vertical and radial epicyclic motions, and this is the lowest and only resonance in the region defined by r_{min} and r_{max}. The associated spin and the radius of the orbit take unique values, which are $a = 0.99616$ and $r = 1.546$. With these parameters fixed the determination of the mass M of the black hole is just a matter of measuring the two frequencies and confirming their 3:1 frequency ratio, and $M = 4603.3/\nu_{up}$, with M measured in units of the sun's mass and ν_{up}, the higher of the two frequencies, in Hz. Of course, harmonics and/or beat frequencies may exist as well (Aschenbach 2004).

3 BLACK HOLES IN ASTROPHYSICS

The existence of astrophysical black holes in the universe has been discussed for quite some time. They have been searched in X-ray binaries with masses typical of stars, and in the cores of galaxies as supermassive black holes with masses exceeding a few million times the mass of the sun and more. On the stellar-mass level promising candidates have been found in low mass X-ray binaries: With a mass significantly exceeding that of the most dense, stable object known, i.e. a neutron star, a black hole is the only candidate for the dark component in the respective binary. About 20 such systems have been detected so far in our Galaxy. The most promising candidate for a supermassive black hole is an object in the centre of the Milky Way, called Sagittarius A* (Sgr A*). In either case the motion of a star, or several stars in case of Sgr A*, orbiting the candidate black hole, is used for the determination of the mass of the compact and dark object through Kepler's laws, i.e. the application of Newtonian physics.

Black holes themselves do not emit. The radiation we receive is generally from gas or dust in the vicinity of the black hole. This matter is orbiting the black hole and may be organized in an accretion disk. Some fraction of this matter may show oscillations and the majority of the scientists tend to associate these with Kepler frequencies. But as shown above the epicyclic frequencies may be equally or even more important.

3.1 The Case of Sgr A*

With the dramatic improvement of the near-infrared capabilities of the large ground-based telescopes over the last ten years and with the launch of the big X-ray observatories Chandra (NASA) and XMM-Newton (ESA) about six years ago the radiation of Sgr A* could be studied in great detail. A major surprise was the discovery of huge flares both in the near-infrared and X-rays. The first report of a quasi-period of about 16.8 min was reported by Genzel et al. (2003). The Fourier analysis of the X-ray flares confirmed this period and revealed additional quasi-periods (Aschenbach et al. 2004). The statistical signifance of the periods per observation is not overwhelming but the fact that they have been observed in more than one independent experiment with excessive power density favours the existence of a true signal. Fig. 4 shows a summary of these frequencies.

Meanwhile later measurements, both in the infrared and X-rays, have supported the presence of quasi-periods in flares. Yusef-Zadeh et al. (2006) have reported a period 33$\pm$2 minutes in HST NICMOS (1.60, 1.87, and 1.90 μm) data, which, within the error bars, agrees very well with the X-ray frequencies of group #1.

From the analysis of an X-ray flare of fairly moderate flux observed with XMM in fall 2004 Bélanger et al. (2005, see also Liu et al. 2006) claim a quasiperiod of 21.4 minutes, which has been incorporated in figure 4 as well (XMM(2)). Looking at just the four data points of group #2 there seems to be a tendency that we observe different frequencies in

those four measurements, although a rigorous error analysis would not exclude a single frequency for group #2. Interestingly, both Genzel el al. (2003) and Liu et al. (2006) claim that the period they found is not constant over the observation but is decreasing from 22.7 minutes to 11.8 minutes (Genzel et al. 2003) and 25 minutes to 17.5 minutes (Liu et al. 2006) based on the separation of the flux maxima/minima. Whether this is actually a decrease of the period remains to be seen, given the fact that just six to nine minima/maxima with fairly low counting statistics and complex lightcurves have been observed. These authors advocate for Keplerian motion with decreasing period because of radial infall with a velocity of about 0.3%×c. Independent of the interpretation these results, because of their short periods - if they are actually orbital periods - underline that the modulation of the flux happens within a few Schwarzschild radii of the black hole and that the Sgr A^* black hole has a significant spin.

Basically, the frequencies can be divided into four groups. The frequencies of the first three groups appear to follow a ratio of 1:2:3, which is confirmed by a best fit.

Because of the appearence of a 3:1 resonance I have associated the highest of the three frequencies with the vertical epicyclic frequency and the lowest of the three frequencies with the radial epicyclic frequency, and I assume that the appearence of these frequencies is due to the new effect of General Relativity described above. In this way the spin a and the orbit radius r are fixed and the mass of the black hole is set by the observed frequencies. For the numerical relation between mass and frequency one may take either the vertical or the radial frequency. Determined in this way it turns out that the mass of the Sgr A^* black hole is $(3.28 \pm 0.13) \times 10^6$ times the mass of our sun. I stress that this mass estimate follows from a straight inverse relation between mass and frequency and does not depend on any other observables like the distance to the black hole. Fig. 5 shows a comparison with the most recent estimates of the Sgr A^* mass which has been derived from more than a decade long dynamical measurements of the orbits of the S-stars around Sgr A^* (Eisenhauer et al. 2005, Ghez et al. 2005). These results depend on the distance to Sgr A^*, and the respective data shown are normalized to the same best estimate of the distance of 7.62 kpc (Eisenhauer et al. 2005). The agreement is more than satisfactory and there is the chance to lower the uncertainty for the QPO measured mass significantly by improving the frequency measurements.

As a side product I mention that these QPOs arise in a region very close to the black hole at $r = 1.546$ or 0.773 Schwarzschild radii, which is an excellent test bed for the effects of strong gravity. For example, the relative difference of the orbital velocity treated by either General Relativity or Newtonian mechanics is more than 65%.

Finally I draw the attention to the frequencies of group #4 in Fig. 4. As spin and mass of the black hole are fixed the Kepler frequency can be calculated as a function of orbital radius. The largest radius considered is the orbit at which the epicyclic oscillations are launched (label r), furtherin I consider the last marginally stable orbit (label ms) and

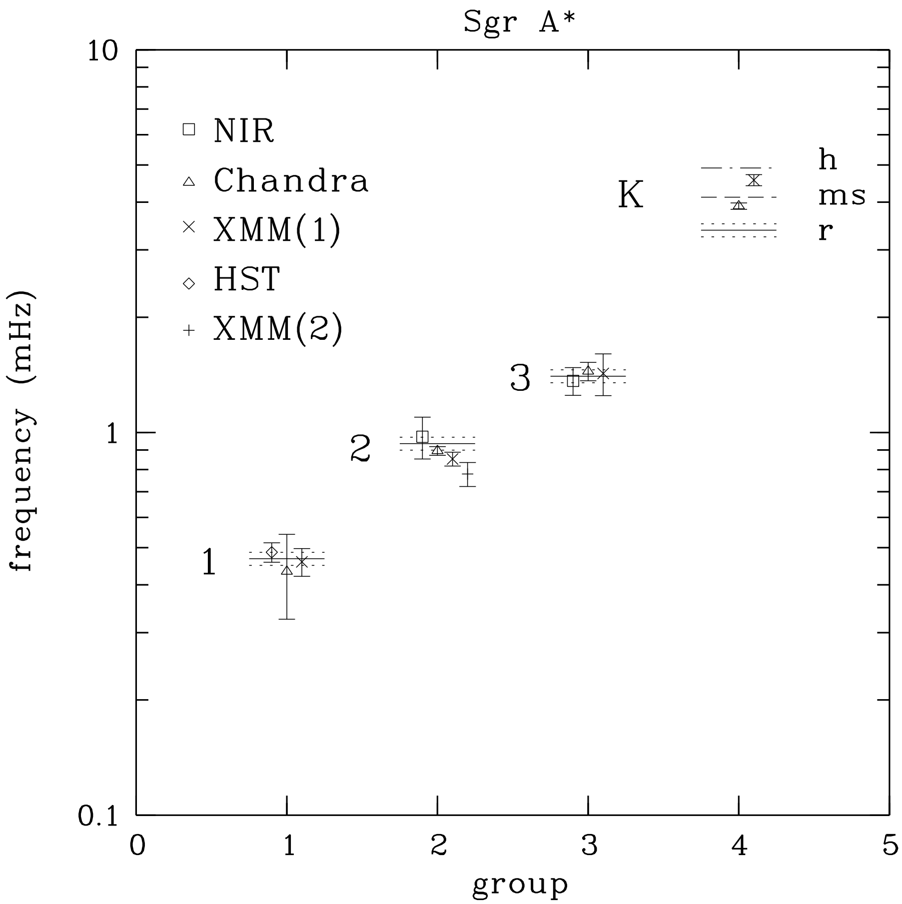

Fig. 4 Quasi-periodic oscillations discovered in at least two independent power density spectra of Sgr A*, in the near-infrared from the ground (NIR, Genzel et al. 2003), X-rays (Chandra and XMM(1), Aschenbach et al. 2004), the infrared with the Hubble Space Telescope (HST, Yusef-Zadeh et al. 2006) and XMM again (XMM(2), Liu et al. 2006; Bélanger et al. 2005). They fall into four groups; the frequencies of the first three groups come in a ratio of 1:2:3. The best fit to such a ratio sequence is indicated by the horizontal solid lines. The $\pm 1\ \sigma$ errors of the best fit are represented by the horizontal dashed lines.

at last the radius of the event horizon (h). The two frequencies observed with Chandra and with XMM-Newton, if confirmed by future measurements, demonstrate that we can in principle see quasi-periodic radiation from below the last stable orbit almost down to the event horizon. Of course these events are likely to be transient and the frequency can vary from one observation to the other between 3.3 and almost 5 mHz depending on the orbital radius. The Kepler frequency may not be strictly periodic, which makes the detection and confirmation difficult and instruments with very large collecting areas exceeding the capabilities of the present generation of telescopes are required.

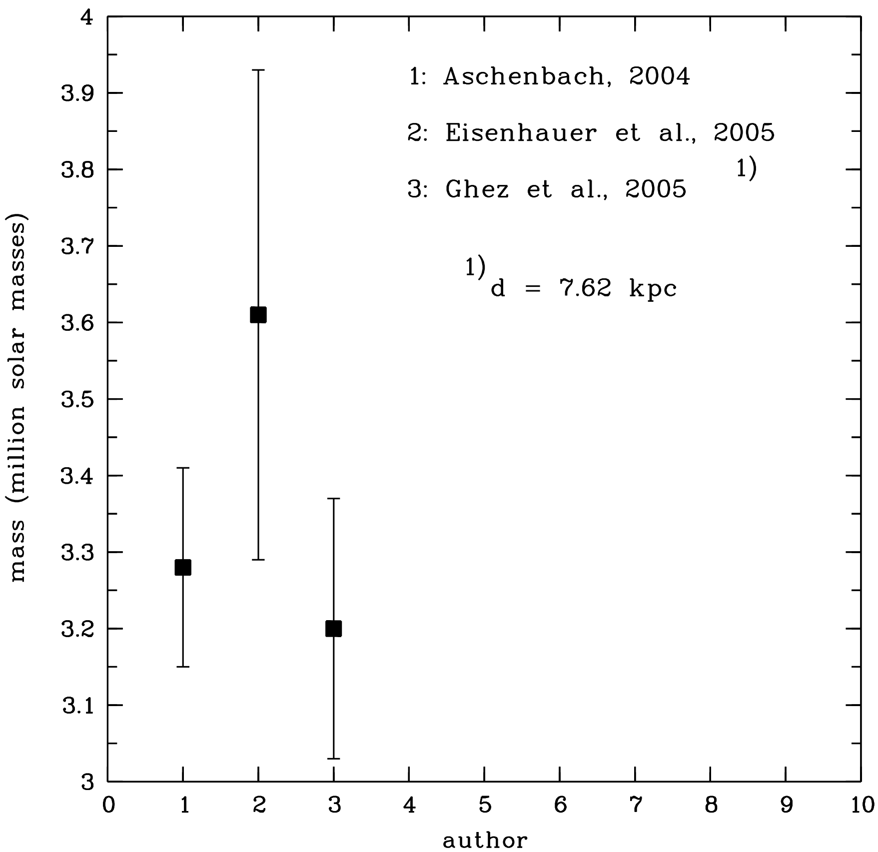

Fig. 5 Mass of the black hole Sgr A* in the centre of our Galaxy. Data 1: mass determined by epicyclic frequencies (Aschenbach 2004), data 2 and 3: mass determined dynamically by measurements of stellar orbits (Eisenhauer et al. 2005, Ghez et al. 2005); these data have been normalized to the best estimate of the distance of d = 7.62 kpc (Eisenhauer et al. 2005).

4 CONCLUSIONS

A new effect concerning the physics of fast spinning Kerr black holes is presented. The relevant range of black hole spin and orbital radii of test particles coincides with regions where a 3:1 resonance between vertical and radial epicyclic frequencies occurs, if the spin is greater than a = 0.9953 and the radius is less than 1.8 gravitational radii. The new effect of a decreasing orbital velocity with decreasing orbital radius may trigger the epicyclic oscillations. Infrared and X-ray observations have indicated the presence of such quasi-periods in flares from Sgr A*, and there is growing evidence that these QPOs are real despite their low statistical significance per observation. If the QPO's, which come in a 1:2:3 ratio, are taken as epicyclic frequencies the mass of the Sgr A* black hole is uniquely determined to $3.3\times10^6 M_\odot$ with an error as big as the frequency measurement error. Independent of the association of frequency with oscillation mode the observed

QPOs demonstrate that the light modulation of the flare emission occurs in the lowest few Schwarzschild radii of the black hole. For the unambiguous determination of the spin of the black hole it will be essential to cover the shortest periods possible. The NIR flare reported by Genzel et al. (2003) shows a separation between two relative maximum flux values as short as 11.8 minutes and the XMM(1) flare goes down to a period of about 219 sec. Of course this needs to be confirmed, but if it is, the observation window of black holes might not only cover the region of the last stable orbit but stretches down close to the event horizon.

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DISCUSSION

DIDIER BARRET: Could you please explain a bit more the reasons to identify the maximum observed frequency with the vertical epicyclic frequency.

ASCHENBACH: For a Kerr BH there are three epicyclic frequencies following the three axes, i.e. orbital (Kepler), polar (vertical) and radial. For any fixed orbital radius and fixed BH spin the Kepler frequency is always greater than the vertical epicyclic frequency, which is always greater than the radial epicyclic frequency. The model I propose assumes a resonance between the vertical and radial frequencies with a ratio of 3:1. If that ratio is observed clearly the higher frequency of the two observed is to be associated with the vertical epicyclic frequency. The Kepler frequency at that orbit radius and that spin is even higher but is nowhere in the orbit in resonance with any one of the other two frequencies.

DIDIER BARRET: How sensitive are your results on the mass with respect to the identification of the QPO frequency?

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ASCHENBACH: In this respect the association of the QPO frequency with a particular oscillation type is absolutely essential. In the relevant equations there are the following four unknowns: the BH mass, the BH spin, the orbital radius at which the oscillation occurs and the oscillation type. To solve that problem unambigously we have to have knowledge or constraints of at least three of the four unknowns. Usually observers assume Kepler orbital motion at the innermost stable orbit to be responsible for the observed frequency which would result in a BH mass BH spin relation. With two frequencies observed the problem is somewhat better defined. The important issue in my work is the discovery of the strange behaviour of the orbital velocity for high values of the spin at low orbits and if this is coupled to the resonant behaviour of the epicyclic frequencies, mass and spin are uniquely determined and the mass is as precise as the frequency measurements are. A resonant behaviour involving the Kepler frequency is definitely excluded for the high spin low orbit case because the resulting BH mass is by far too low compared with the dynamical mass.

DANIELE FARGION: Does General Relativity explain jets?

ASCHENBACH: I have no straight answer to that, but I think one should not rule out such a possibility (c.f. Aschenbach 2004). If the vertical epicyclic oscillations exist and are created close to the inner edge of the accretion disk, why shouldn't mass and energy be expelled by them perpendicular to the accretion disk. This definitely needs further study.

DANIELE FARGION: Did you consider jets drag on your frequency?

ASCHENBACH: No, I did not.

WOLFGANG KUNDT: Congratulations on your findings. But when discussing the alternative possibilities you did not mention the Supermassive Magnetized Disk (SMD) / burning disk model (e.g. W. Kundt in Astrophysics and Space Science, 62, 335 (1979)).

ASCHENBACH: In the introduction of this talk I have tried to summarize the arguments that have led people to state "the most convincing case for the existence of an astrophysical black hole is Sgr A*". This statement has been made by various scientists, for instance, at the Joint Astronomy Conference 'Growing Black Holes: Accretion in a Cosmological Context' held at Garching in June 2004. The general consensus seems to be that we are still lacking 'the proof' for Sgr A* being a black hole. Therefore other models are still something to persue. In my talk I used the physical properties expected for black holes and used them to understand the QPOs, and as far as I am concerned I don't see any contradiction, which does not tell that they are eventually nothing more than a viable model.

N̲ANDA REA: Do you expect any spectral variability during these oscillations?

ASCHENBACH: If strong gravity effects produce the light variations as Marek Abramowicz and others have recently suggested I would expect some spectral variations.

N̲ANDA REA: Did you check for spectral variations doing a phase resolved spectroscopy during the XMM and Chandra flares?

ASCHENBACH: Yes we did that, but we did not find any significant variations. But I stress that the relative flux of the modulated signal is very low. X-ray telescopes with much larger collecting area are needed.